647510 **MSC1MatC102x** **Seat No :________**

M.Sc.(Maths.) Semester - 1 (*CBCS*) Examination
OCT/NOV-2024
Real Analysis(Core (New))

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1(A) Answer any two out of three **(10)**

(1) Show that countable union of measurable sets is again measurable.

(2) Prove that every closed and open set are measurable.

(3) Prove that if f and g are measurable functions then $f \cdot g$ is also measurable function.

Que.1(B) Answer any one out of two **(04)**

(1) Define Lebesgue outer measure of set and show that Lebesgue outer measure of a finite interval is its length.

(2) Show that every step function is measurable.

Que.2(A) Answer any two out of three **(10)**

(1) Show that $[a, b]$ is uncountable.

(2) State and prove Fatou's lemma.

(3) State and prove Egoroff's theorem.

Que.2(B) Answer any one out of two **(04)**

(1) State and prove monotone convergence theorem.

(2) Prove that Cantor set is measurable and hence find its measure.

Que.3(A) Answer any two out of three **(10)**

(1) If f is function of bounded variation on $[a, b]$ then $T_a^b = P_a^b + N_a^b$ and $f(b) - f(a) = P_a^b - N_a^b$

(2) A function f is of bounded variation iff f is difference of two monotonically real valued increasing functions on $[a, b]$.

(3) If function f is integrable on $[a, b]$ and $\int_a^b f(t)dt = 0$ for $x \in [a, b]$ then show that $f = 0$ almost everywhere in $[a, b]$.

Que.3(B) Answer any one out of two **(04)**

(1) If function f is bounded away from zero then $\frac{1}{f}$ is of bounded variation.

(2) Prove that the finite union of measurable sets is measurable.

Que.4(A) Answer any two out of three **(10)**

(1) Show that $\|.\|_1$ is normed space on S_1 / R & pair $(S_1/R , \|.\|_1)$ is normed linear space over $\mathbb{R}$ which is denoted by $L^1[0,1]$.

(2) Let f be integrable function on $[a, b]$. And let $F(x) = F(a) + \int_a^b f$ for all $x \in [a, b]$ then $F'(x) = f(x)$ for almost all $x \in [a, b]$.

(3) State and prove Minkowski's Inequality.

Que.4(B) Answer any one out of two **(04)**

(1) Let X be a non-empty subset of $\mathbb{R}$ and define $d: X \times X \to \mathbb{R}$ by $d(x, y) = \|x - y\|, \forall x, y \in X$ then show that d is a metric on $\mathbb{R}$ and (X, d) is metric space.

(2) Define norm of a vector space and Pseudo norm (P-norm).

Que.5(A) Answer any two out of three **(10)**

(1) If function f is an absolutely continuous on $[a, b]$ and $f' = 0$ a.e. then f is constant function.

(2) State and prove the Lebesgue's Dominated Convergence theorem.

(3) Let f bean integral function on $[a, b]$. Let $F(x) = F(a) + \int_a^x f(t)dt$ for all $x \in [a, b]$. Then prove that $F'(x) = f(x)$ for almost all $x \in [a, b]$.

Que.5(B) Answer any one out of two **(04)**

(1) Show that if E is measurable set then its complement is also measurable.

(2) Let E_n be an infinite decreasing sequence of measurable sets $E_{n+1} \subset E_n$ for each n, with $m(E_1)$ is finite. Then prove that $m(\cap_{k=1}^{\infty} E_k) = \lim_{n\to\infty} m(E_n)$.

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